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High Speed Transport Cruise Drag

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HIGH SPEED TRANSPORT CRUISE DRAG

Abstract

This report provides scaling laws for the cruise aerodynamics of high speed transport wings based on the results of Navier-Stokes computations. Expressions for the various drag components are found, together with the corresponding value of $\left(\frac{L}{D}\right)_m$ for various values of the geometric parameter $\frac{s}{l}$ which allow for simple optimization of the wing configurations with respect to span. It is found that linear theory expressions can be used for this purpose provided the coefficients of these expressions for C_D and $\left(\frac{L}{D}\right)_m$ are available using Navier Stokes results.

List of Symbols

C_D	drag coefficient
$C_{D,0}$	drag coefficient at zero lift
C_{DF}	friction coefficient
C_L	lift coefficient
$(C_L)_m$	lift coefficient at maximum L/D
L/D	lift to drag ratio
$(L/D)_m$	maximum lift to drag ratio
$(L/D)_{m,\max}$	maximum lift to drag ratio with respect to $\frac{s}{l}$
$\frac{s}{l}$	span-to-length ratio
S	area
$p = \frac{S}{2sl}$	shape parameter

$\bar{p}$ mean value

$\tau = \frac{\text{vol}}{S^{\frac{3}{2}}}$ volume coefficient (= .039)

Introduction

An improved understanding of the cruise aerodynamics of the high speed transport wing requires that both theoretical and experimental methods be applied to define alternative configurations suitable for the high speed flight regime. Previous aerodynamic analysis has relied heavily on linear analysis together with extensive testing of cruise configurations. In order to prepare for future designs new methods of analysis utilizing modern CFD techniques appropriately validated by wind tunnel experiments must be applied before configurations are selected for experimental development in large facilities.

The approach used for this report utilizes the results of Navier Stokes computations for variations primarily in $\frac{s}{l}$ about a Boeing wing used as a baseline. The lift and drag increments are assumed to have the same dependence on the planform parameters as suggested by linear theory but the dimensionless coefficients are now evaluated using the non-linear Navier-Stokes results. In this way a simple general expression for L/D for the configurations can be formulated.

Computations have been made to demonstrate the approach and results are given for $(L/D)_m$ permitting a reduction in the experimental testing required. An upper limit to $(L/D)_m$ is found and the corresponding optimum value of $\frac{s}{l}$ is given.

Analysis

Linear theory suggests that the drag coefficient of a supersonic configuration can be written in the form

$$C_D = C_{DF} + \frac{512}{\pi}\tau^2 p^2 \left(\frac{s}{l}\right)^2 K_o + \frac{1}{2\pi} C_L^2 \frac{p}{\left(\frac{s}{l}\right)} \left[K_w + 2\beta^2 K_m \left(\frac{s}{l}\right)^2\right] \quad (1)$$

where

C_{DF} is associated with skin friction drag

K_o is associated with wave drag due to thickness

K_v is associated with vortex drag

K_w is associated with wave drag due to lift

C_L is the lift coefficient

and

$\beta^2 = M_\infty^2 - 1$

In the Navier-Stokes computations $\frac{s}{l}$ was varied by a scale factor f from a nominal of $\frac{s}{l} = .4336895$, ie $\frac{s}{l} = .4336895$ f, $1.25 >$ f $>$ .625, and p was varied from a nominal value of p = .2332929 to a value of $p = .2210521$

A general equation for the drag coefficient was postulated of the form

$$C_D = A + Bp^2 f^2 + \frac{C_L^2 p}{f}(C + Df^2) \tag{2}$$

where A,B,C and D are coefficients to be evaluated using the Navier Stokes results and a mean value of p given as $\bar{p} = .2281858$, is used. The expression (2) is analogous to (1) and is useful only if the value of C_D that results is sufficiently close to that given by the Navier-Stokes results.

(1) Zero Lift

First mean value of the drag at zero lift were found from the Navier-Stokes computations for each value of f:

$$C_{D_o} = .008546 \text{ (mean of wings 15, 25, 35 and 42; } f = 1.25)$$

$$= .007541 \text{ (mean of wings 1, 2, 3 and 4; } f = 1)$$

$$= .006700 \text{ (mean of wings 16, 21, 31, and +3; } f = .875)$$

$$= .005832 \text{ (mean of wings 12, 22, 32, and 44; } f = .75)$$

$$= 005176 \text{ (mean of wings 13, 23, 33, and 45} f = .625)$$

and the constant A and B evaluated by matching the results for f = 1 and f = .625; this gives

$$A = .003660 \text{ and } B = .074537$$

Thus

$$C_{D_o} = .003660 + .074537 p^2 f^2 \tag{3}$$

Table 1 shows the values of C_{D0} computed, for each wing, by the Navier Stokes equations with the result given by equation (3).

Table 1 - C_{D0}

Navier-Stokes		**Mean**	**Equation 3**
Wing 15	.008854	.008846	.008572
Wing 25	.008551		
Wing 35	.008636		
Wing 42	.008342		
Wing 1	.007552	.007541	.007541
Wing 2	.007734		
Wing 3	.007478		

Wing 4	.007402		
Wing 16	.006851	.006670	.0066631
Wing 21	.006144		
Wing 31	.006608		
Wing 43	.006577		
Wing 12	.005799	.005532	.005843
Wing 22	.005845		
Wing 32	.005851		
Wing 44	.005833		
Wing 13	.005151	.005176	.005176
Wing 23	.005168		
Wing 33	.005180		
Wing 45	.005205		

(2) Non Zero Lift

Similarly, the mean values of the quantity $\frac{(C_D - C_{D0})}{C_L^2}\frac{f}{p}$ were found from the Navier Stokes computations at the value of C_D corresponding to that for the maximum computed value of (L/D); these values at $f = 1$ and $f = .625$ were used to find $CandD$ $\left(\text{since } \frac{C_D - C_{D0}}{C_L^2}\frac{f}{p} = C + Df^2\right)$ thus

$$\frac{C_D - C_{D0}}{C_L}\frac{f}{p} = 1.95060 \text{ (mean of wings 15, 25, 35 and 42; } f = 1.25)$$

$$= 2.6683 \text{ (mean of wings 1, 2, 3 and 4; } f = 1.0)$$

$$= 2.34708 \text{ (mean of wings 16, 21, 31 and 43; } f = .875)$$

$$= 2.08039 \text{ (mean of wings 12, 22, 32 and 44; } f = .75)$$

$$= 1.81567 \text{ (mean of wings 13, 23, 33 and 45; } f = .625$$

and the constants C and D evaluated by matching the results for f = 1 and f = .625; this gives

$$C = 1.2691 \qquad D = 1.3992$$

so that

$$\frac{C_D - C_{D0}}{C_L^2}\frac{f}{p} = 1.2691 + 1.3992 f^2 \tag{4}$$

Table II shows the value of $\frac{C_D - C_{D0}}{C_L^2}\frac{f}{p}$ computed for each wing by the Navier-Stokes equations with the result given by equation (4).

Table II - $\frac{C_D - C_{D0}}{C_L^2}\frac{f}{p}$

Navier-Stokes		**Mean**	**Equation 4**
Wing 15	2.88637	2.95060	3.0399
Wing 25	2.96967		
Wing 35	2.97739		
Wing 42	2.96598		
Wing 1	2.74649	2.66834	2.6683
Wing 2	2.59041		
Wing 3	2.66748		
Wing 4	2.66903		
Wing 16	2.28755	2.34708	2.34036
Wing 21	2.35686		
Wing 31	2.37203		
Wing 43	2.37183		

Wing 12	2.08080	2.08039	2.05615
Wing 22	2.07449		
Wing 32	2.08409		
Wing 44	2.09281		
Wing 12	1.80017	1.81567	1.81567
Wing 23	1.80800		
Wing 33	1.82395		
Wing 45	1.83057		

Total Drag

The expression for total drag is therefore

$$C_D = .003660 + .074537p^2f^2 + \frac{C_L^2 p}{f}\left[1.2691 + 1.3992f^2\right] \tag{5}$$

Comparing (1) and (2) it can be seen that

$$K_o = \frac{B}{\tau^2}\left(\frac{f}{\frac{s}{l}}\right)^2 \frac{\pi}{512} = 1.5989$$

$$K_v = 2\pi\left(\frac{\frac{s}{l}}{f}\right) C = 3.4582$$

and

$$K_w = \frac{\pi}{B^2}\left(\frac{f}{\frac{s}{l}}\right) D = 2.1293$$

for $\beta^2 = (2.4^2 - 1)$

Thus good agreement is found between equation (1) and the Navier Stokes results if (with $\tau = .039$)

$$K_o = 1.598;\ K_v = 3.4582$$

and

$$K_w = 2.1293.$$

Equation (1) can then be used in the analysis of the aerodynamics of the wing.

With C_D in the simple form the maximum value of L/D for given f is found by varying C_L and is

$$\left(\frac{L}{D}\right)_m = \frac{1}{2}\left[\frac{(A + Bp^2f^2)(C + Df^2)p}{f}\right]^{-\frac{1}{2}} \tag{6}$$

which occurs at a lift coefficient C_L given by

$$(C_L))_m = \left[\frac{(A + Bp^2f^2)p}{f(C + Df^2)}\right]^{\frac{1}{2}}$$

Figure 1 shows $\left(\frac{L}{D}\right)_m$ as a function of f for the family of configurations.

An optimum value of f can be found at which $\left(\frac{L}{D}\right)_m$ is a maximum by differentiation of 6 with respect to f, thus the optimum occurs at f given by

$$\frac{2f}{f^2 + a} + \frac{2f}{f^2 + c} - \frac{1}{f} = 0$$

when $a = \frac{A}{Bp^2}$, $c = \frac{C}{D}$ which can be solved for f_{opt} to give

$$\begin{aligned} f_{\text{opt}} &= \frac{1}{\sqrt{6}}\left[\left(a^2 + c^2 + 14ac\right)^{\frac{1}{2}} - (a + c)\right]^{\frac{1}{2}} \\ &= .5552 \text{ for } p = .2281858 \end{aligned}$$

Thus the optimum value of f is substantially less than 1 (the baseline value) and corresponds to a value of $\frac{s}{l} = .5552 \times .4336895 = .2408$.

If $\frac{s}{l} >> .2408$ the wave drag due to thickness (which varies as $\left(\frac{s}{l}\right)^2$) and the wave drag due to lift (which varies as $\left(\frac{s}{l}\right)$ are too large. If $\frac{s}{l} << .2408$ the vortex drag (which varies as $\left(\frac{s}{l}\right)^{-1}$) is too large.

The value of $\left(\frac{L}{D}\right)_m$ with $f = .5552$ and $p = .2281858$ is given by equation (6) as

$$\left(\frac{L}{D}\right)_{m,\max} = 8.583$$

This is the maximum value of $\left(\frac{L}{D}\right)_m$ that can be attained by this family of wings (and compares with $\left(\frac{L}{D}\right)_m = 7.405$ for $f = 1$).

Concluding Remarks

It is found that the expression for supersonic cruise drag of a wing given by linear theory can be used and the components of drag identified by evaluating certain coefficients in this expression using the results of Navier-Stokes computation. This then facilitates the calculation of the maximum $\frac{L}{D}$ attainable by this family of wings and the optimum $\frac{s}{l}$ at which this occurs.

Thus it is found that the supersonic cruise drag can be expressed as

$$C_D = C_{DF} + \frac{512}{\pi}\tau^2 p^2 \left(\frac{s}{l}\right)^2 K + \frac{1}{2\pi} C_L^2 \frac{p}{\left(\frac{s}{l}\right)} \left[K_v + 2K_w \beta^2 \left(\frac{s}{l}\right)^2\right]$$

where K_o, K_V and K_w are evaluated as $K_o = 1.5987; K_m = 3.458$ and $K_w = 2.129$.

The maximum value of $\left(\frac{L}{D}\right)_m = 8.58$ was obtained at a value of $\frac{s}{l} = .2408$ suggesting that a smaller value of $\frac{s}{l}$ than the nominal value of .4337 gives improved performance.

References

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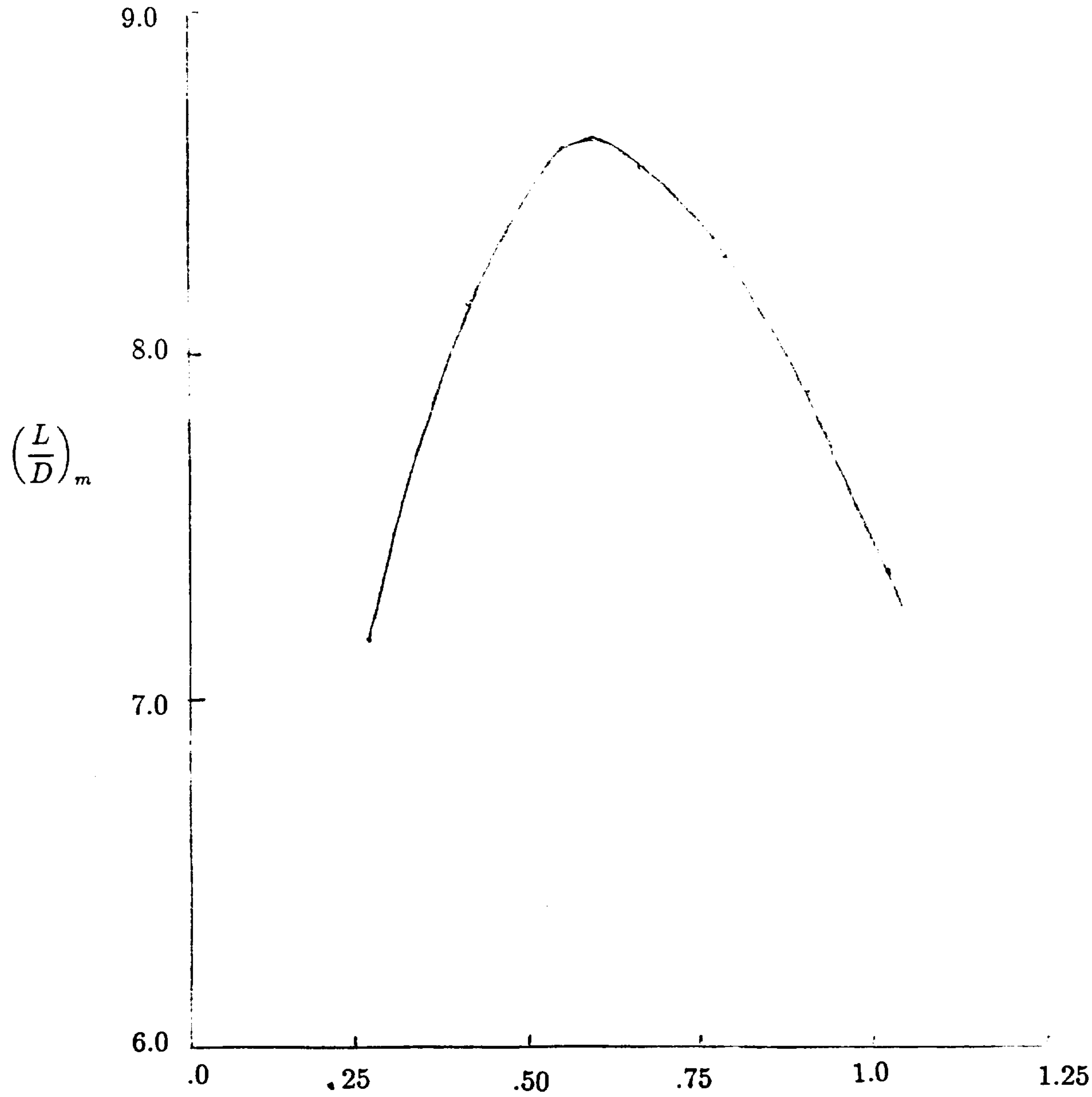

Fig. 1 $\left(\frac{L}{D}\right)_m$ as a function of f

for $\bar{p} = .228186, \left(\frac{s}{l}\right)_{\text{nom}} = .4336895$